**Chap 3: Z Transform**

1. **The Z Transform**
2. **Z Transform**

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| **Definition** | * Causal sequence: * Infinite sequence: * The z Transform of an **infinite** **sequence** is defined whenever the sum exists and where is a complex variable: * The Z Transform of a **causal sequence**:   + **:** is the z transform operator   + **:** is a z transform pair |
| Example | Determine the z transform? |
| **Z Transform Table** | |  |  |  | | --- | --- | --- | |  |  | **Region of existence** | | Unit Impulse Sequence |  |  | | Unit Step Sequence |  |  | | (a constant) |  |  | |  |  |  | | (a constant) |  |  | | (T constant) |  |  | | (T,w constant) |  |  | | (T,w constant) |  |  | | (w constant) |  |  | | (w constant) |  |  | |
| **Properties** | |  |  | | --- | --- | |  | | | Linearity |  | | First shift theorem – Delaying |  | | Second shift theorem – Advancing |  | | Multiplication by | If then with the constant | | Multiplication by | If then with a positive integer | | Initial-value theorem | If is a sequence with z transform then the initial-value theorem states that | | Final-value theorem | If is a sequence with z transform then the final-value theorem states that | |
| Example | * We have: * So: * We have: * So: * We have: * So: * We have: * So: * We have: * So: * We have: * So: * We have: * So: |

1. **Inverse Z Transform**

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| **Inverse Z Transform** | * If we have then:   + inverse z-transform operator |
| Example | **Dạng 1 – Easy & Simple**   * Let: * Taking the inverse z transform: |
| * Let: * Use the first shift theorem, we take the inverse z transform: * And: * Therefore: * Let: * Use the first shift theorem, we take the inverse z transform: * And: * Therefore: |
| * The denominator of the transform may be factorized as: * We have: * We know that: * Therefore: |

1. **Discrete-time Systems & Difference Equations**
2. **Difference equations**

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| **Difference equations** | * The main use for Z-Transforms is solving Difference Equations | |
| Find the difference equation of     * To analyse the system, let denote the sequence of input signals to D; then, owing to the delay action of D, we have: | |
| * Also, owing to the feedback action: * So the difference equation is: |  |
| Find the difference equation of     * Let and denote the sequence of input signals to D; then, owing to the delay action of D, we have: * Also, owing to the feedback action: * We have: * So the difference equation is: | |
| **. Draw a block diagram?**  **Note that we should rearrange the equation with the form:**   * We have: * Because this is a second order equation so there are 2 block D      * Then adding the sum junction: | |

1. **Solving Difference Equations**

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| **Solving Difference equations** | * We have: * Taking Z transforms, we have: * Using 2nd shift theorem: * Taking the inverse Z transform: |
| * Taking Z transforms, we have: * Using 2nd shift theorem: * Taking the inverse Z transform: * Taking Z transforms, we have: * Using 2nd shift theorem: * Taking the inverse Z transform: |

1. **Discrete linear systems**

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| **Z transfer functions** | **Definition:**  The **Z transfer function** of a linear time-invariant system under the assumption that **all the initial conditions are 0** (the system is **in a quiescent state**)  **How to find the Z Transfer Function?**   * If we have a linear time-invariant system characterized by the difference equation: * Then we obtain the Laplace Transform with **all the initial conditions are 0**: * Then we obtain the **Z Transfer Function**   **Characteristic:**   * The transfer function may be expressed as * is the **characteristic equation** of the discrete system   + Its order determines the **order** of the system   + Its roots are referred to as the **poles** of the discrete system * The root of are referred to as the **zeros** of the discrete system |
| Example | . Find the Z transfer function?   * Taking the Z transform with all initial conditions are 0 * Using 2nd shift theorem: * The Z transfer function: |
| **The impulse response** |  |
| Example |  |
| **Stability** |  |
| Example |  |
| **Convolution** |  |
| Example |  |

**IV. The relationship between Laplace & z transforms**

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